COURSE TITLE: MATERIAL FOR ELECTRICAL ENGINEERING

Lesson II: EINSTEINS AND PLANCKS PHOTO-ELECTRIC EQUATIONS

I.1 Modelling with particles and waves

In this chapter, we will study two very powerful scientific models – particles and waves – to see how they can help us to understand more about both light and matter. First we will take a closer look at each of these models in turn.

I.1.1 Particle Models

In order to explain the properties of matter, we often think about the particles of which it is made and the ways in which they behave. We imagine particles as being objects that are hard, have mass and move about according to the laws of Newtonian mechanics(See figure 1). When two particles collide, we can predict how they will move after the collision, based on knowledge of their masses and velocities before the collision. If you have played snooker or pool, you will have a pretty good idea of how particles behave.

Particles are a macroscopic model. Our ideas of particles come from what we observe on a macroscopic scale – when we are walking down the street, or observing the motion of stars and planets, or working with trolleys and balls in the laboratory. But what else can we explain using a particle model?

The importance of particle models is that we can apply them to the microscopic world, and explain more phenomena.

Figure 1: Particles in motion

Table1 below shows how, in particular areas of science, we can use a particle model to interpret and make predictions about macroscopic phenomena.

Table1: Particles models in science

Area	**Model**	Macroscopic phenomena
Electricity	Flow of electrons	Current
Gases	Kinetic theory	Pressure, temperature and volume of a gas
Solids	Crystalline materials	Mechanical properties
Radioactivity	Nuclear model of the atom	Radioactive decay, fission and fusion reactions
Chemistry	Atomic Structure	Chemical reactions

I.1.2 Wave models

Waves are something that we see on the sea. There are tidal waves, and little ripples. Some waves have foamy tops, others are breaking on the beach.

Physicists have an idealised picture of a wave – it is shaped like a sine graph. You will not see any waves quite this shape on the sea. However, it is a useful picture, because it can be used to represent some simple phenomena. More complicated waves can be made up of several simple waves, and physicists can cope with the mathematics of sine waves.

Waves are a way in which energy is transferred from one place to another. In any wave, something is changing in a regular way, while energy is travelling along. In water waves, the surface of the water moves up and down periodically, and energy is transferred horizontally.

Table2 shows some phenomena that we explain in terms of waves.

Table2: Wave models in science

Phenomenon	**Varying quantity**
Sound	Pressure(or density)
Light(and other electromagnetic waves)	Electric and magnetic field strenghts
Waves on strings	Displacement

The characteristic properties of waves are that they all show reflection, refraction, diffraction and interference. Waves themselves do not have mass or charge. Since

particle models can also explain reflexion and refraction, it is diffraction and interference that we regard as defining the characteristics of waves. If we can show diffraction and interference, we know that we are dealing with waves.

I.1.3 Waves or Particles

Wave models and particle models are both very useful. They can explain a great many different observations. But which should we use in a particular situation? And what if both models seem to work when we are trying to explain something?

This is just the problem that physicists struggled with for over a century, in connection with light. Does light travel as a wave or as particles?

For a long time, Newton's view prevailed – light travels as particles. This was set out in 1704 in his famous book Opticks. He could use this model to explain both reflection and refraction. His model suggested that light travels faster in water than in air. In 1801 Thomas Young, an English physicist, demonstrated that light showed diffraction and interference effects. Physicists were still very reluctant to abandon Newton's particle model of light. The ultimate blow to Newton's model came from the work carried out by the French physicist Léon Foucault in 1853. His experiments on the speed of light showed that light travelled more slowly in water than in air. Newton's model was in direct contradiction with experimental results. Most scientists became convinced that light travelled through space as a wave.

I.2 Particulate Nature of Light

We expect light to behave as waves, but can light also behave as particles? The answer is yes, and you are probably already familiar with some of the evidence. If you place a Geiger counter next to a source of gamma radiation you will hear an irregular series of clicks. The counter is detecting γ-rays (gamma-rays). But γ-rays are part of the electromagnetic spectrum. They belong to the same family of waves as visible light, radio waves, X-rays, etc.

So, here are waves giving individual or discrete clicks, which are indistinguishable from the clicks given by α-particles (alpha-particles) and β-particles (beta- particles). We can conclude that γ-rays behave like particles when they interact with a Geiger counter.

This effect is most obvious with γ-rays, because they are at the most energetic end of the electromagnetic spectrum. It is harder to show the same effect for visible light.

I.2.1 Photons

The photoelectric effect, and Einstein's explanation of it, convinced physicists that light could behave as a stream of particles. Before we go on to look at this in detail, we need to see how to calculate the energy of photons.

Newton used the word corpuscle for the particles which he thought made up light. Nowadays, we call them photons and we believe that all electromagnetic radiation consists of photons. A photon is a 'packet of energy' or a quantum of electromagnetic energy. Gamma-photons (γ-photons) are the most energetic. According to Albert Einstein, who based his ideas on the work of another German physicist, Max Planck, the energy E of a photon in joules (J) is related to the frequency f in hertz (Hz) of the electromagnetic radiation of which it is part, by the equation:

$$\mathbf{E = hf}$$

The constant h has an experimental value equal to **6.63 × 10−3 4 J s**.

This constant h is called the Planck constant. It has units of joule seconds (J s), but you may prefer to think of this as ' joules per hertz'. The energy of a photon is directly proportional to the frequency of the electromagnetic waves, that is:

$$\mathbf{E \propto f}$$

Hence, high-frequency radiation means high-energy photons.

Notice that the equation **E = hf** tells us the relationship between a particle property (the photon energy E) and a wave property (the frequency f). It is called the Einstein relation and applies to all electromagnetic waves.

The frequency f and wavelength λ of an electromagnetic wave are related to the wave speed c by the wave equation $c = f\lambda$, so we can also write this equation as:

$$\mathbf{E = hc/\lambda}$$

It is worth noting that the energy of the photon is inversely proportional to the wavelength. Hence the short- wavelength X-ray photon is far more energetic than the long-wavelength photon of light.

Examples

1- Calculate the energy of a high-energy γ-photon, of frequency 10   Hz.

2- Visible light has wavelengths in the range 400 nm (violet) to 700 nm (red). Calculate the energy of a photon of red light and a photon of violet light.

Datas: speed of light in a vacuum c = 3.00 × 10 m s

Planck constant h = 6.63 × 10−3 4 J s

Now we can work out the energy of a γ-photon. Gamma-rays typically have frequencies greater than 10^{20} Hz. The energy of a γ-photon is therefore greater than $(6.63 \times 10^{-34} \times 10^{20}) \approx 10^{-13}$ J. This is a very small amount of energy on the human scale, so we don't notice the effects of induvidual γ-photons. However, some astraunouts have reported seeings flashes of light as individual cosmic rays, high-energy γ-photons, passed through their eyeballs.

I.2.2 The electronvolt (eV)

The energy of a photon is extremely small and far less than a joule. Hence the joule is not a very convenient unit for measuring photon energies. You may remember the electronvolt, when considering amounts of energy much smaller than a joule.

When an electron travels through a potential difference, energy is transferred. If an electron, which has a charge of magnitude 1.60×10^{-19} C, travels through a potential difference of 1V, its energy change W is given by:

$W = QV = 1.60 \times 10^{-19} \times 1 = 1.60 \times 10^{-19}$ J

Hint: One electronvolt (1 eV) is the energy transferred when an electron travels through a potential difference of one volt.

Therefore:

$1 \text{ eV} = 1.60 \times 10^{-19}$ J

So when an electron moves through 1 V, 1 eV of energy is transferred. When one electron moves through 2 V, 2 eV of energy is transferred. When five electrons move through 10 V, a total of 50 eV is transferred, and so on.

- To convert from eV to J, multiply by 1.60×10^{-19}.
- To convert from J to eV, divide by 1.60×10^{-19}.

When a charged particle is accelerated through a potential difference V, its kinetic energy increases. For an electron (charge e), accelerated from rest, we can write:

$eV = 1/2(mv^2)$

We need to be careful when using this equation. It does not apply when a charged particle is accelerated through a large voltage to speeds approaching the speed of light c. For this, we would have to take account of relativistic effects. (The mass of a particle increases as its speed gets closer to 3.00×10^8 m/s.)

Rearranging the equation gives the electron's speed:

$$\mathbf{v} = \sqrt{\frac{2eV}{m}}$$

This equation applies to any type of charged particle, including protons (charge +e) and ions.

I.2.3 Estimating the Planck constant

You can obtain an estimate of the value of the Planck constant h by means of a simple experiment. It makes use of light-emitting diodes (LEDs) of different colours (Figure 2). LED conducts in one direction only (the forward direction) and that it requires a minimum voltage, the threshold voltage, to be applied in this direction before it allows a current. This experiment makes use of the fact that LEDs of different colours require different threshold voltages before they conduct and emit light.

- A red LED emits photons that are of low energy. It requires a low threshold voltage to make it conduct.
- A blue LED emits higher-energy photons, and requires a higher threshold voltage to make it conduct.

What is happening to produce photons of light when an LED conducts? The simplest way to think of this is to say that the electrical energy lost by a single electron passing through the diode reappears as the energy of a single photon.

Figure 2: Light-emitting diodes (LEDs) come in different colours. Blue (on the right) proved the trickiest to develop.

Hence we can write:

energy lost by electron = energy of photon

$$\mathbf{eV=(hc/\lambda)}$$

where V is the threshold voltage for the LED. The values of e and c are known. Measurements of V and λ will allow you to calculate h. So the measurements required are:

- V – the voltage across the LED when it begins to conduct (its threshold voltage). It is found using a circuit like the one shown in Figure 30.6a
- λ – the wavelength of the light emitted by the LED. This is found by measurements using a diffraction grating or from the wavelength quoted by the manufacturer of the LED.

If several LEDs of different colours are available, V and λ can be determined for each and a graph of V against 1/λ drawn(Figure 4). The gradien of this graph will be **(hc)/e** and hence h can be estimated.

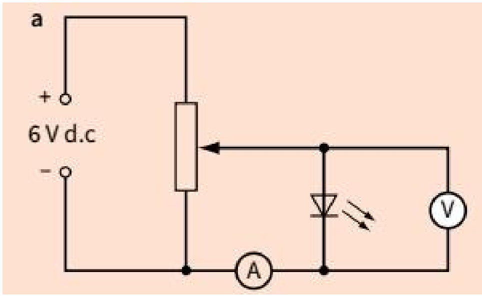

Figure 3: A circuit to determine the threshold voltage required to make an LED conduct. An ammeter helps to show when this occurs

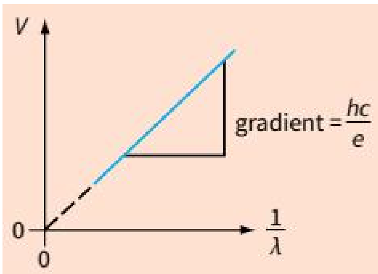

Figure 4: The graph used to determine h from this experiment.

I.3 Photoelectric effect

In the photoelectric effect, light shines on a metal surface and electrons are released from it. The Greek word for light is photo, hence the word ‘photoelectric’. The electrons removed from the metal plate in this manner are often known as photoelectrons.

The apparatus used to observe the photoelectric effect is shown in the section below. Light from a lamp is shone onto a negatively charged metal plate and some of the electrons in the metal are emitted. A simple explanation is that light is a wave that

carries energy and this energy releases electrons from the metal. However, detailed observations of the effect at first proved difficult to explain, in particular that there is a minimum threshold frequency of light below which no effect is observed.

I.3.1 Observing the Photoelectric effect

You can observe the photoelectric effect yourself by fixing a clean zinc plate to the top of a gold-leaf electroscope (Figure 5). Give the electroscope a negative charge and the leaf deflects. Now shine electromagnetic radiation from a mercury discharge lamp on the zinc and the leaf gradually falls. (A mercury lamp strongly emits ultraviolet radiation.) Charging the electroscope gives it an excess of electrons. Somehow, the electromagnetic radiation from the mercury lamp helps electrons to escape from the surface of the metal.

Placing the mercury lamp closer causes the leaf to fall more rapidly. This is not very surprising. However, if you insert a sheet of glass between the lamp and the zinc, the radiation from the lamp is no longer effective. The gold leaf does not fall. Glass absorbs ultraviolet radiation and it is this component of the radiation from the lamp that is effective.

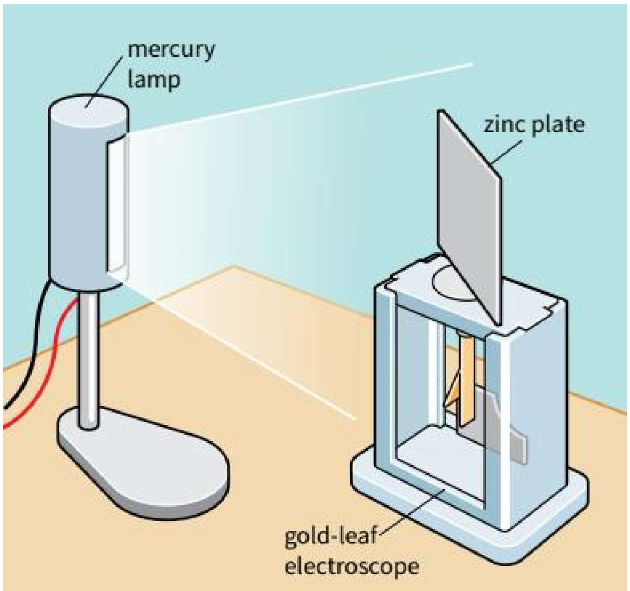

Figure 5: A simple experiment to observe the photoelectric effect.

I.3.2 Low Frequency, High Frequency

If you try the experiment described above with a bright filament lamp, you will find it has no effect. It does not produce ultraviolet radiation. There is a minimum frequency

that the incident radiation must have in order to release electrons from the metal. This is called the threshold frequency. The threshold frequency is a property of the metal plate being exposed to electromagnetic radiation.

Hint: The threshold frequency is defined as the minimum frequency required to release electrons from the surface of a metal.

Physicists found it hard to explain why weak ultraviolet radiation could have an immediate effect on the electrons in the metal, but very bright light of lower frequency had no effect. They imagined light waves arriving at the metal, spread out over its surface, and they could not see how weak ultraviolet waves could be more effective than the intense visible waves. In 1905, Albert Einstein came up with an explanation based on the idea of photons.

Metals (such as zinc) have electrons that are not very tightly held within the metal. These are the conduction electrons, and they are free to move about within the metal. When photons of electromagnetic radiation strike the metal, some electrons break free from the surface of the metal (Figure 6). They only need a small amount of energy (about 10^-19 J) to escape from the metal surface.

We can picture the electrons as being trapped in an energy 'well' (Figure 7). A single electron requires a minimum energy Φ (Greek letter phi) to escape the surface of the metal. The **work function energy**, or **simply work function**, of a metal is the minimum amount of energy required by an electron to escape its surface. (Energy is needed to release the surface electrons because they are attracted by the electrostatic forces due to the positive
met a l ions .)

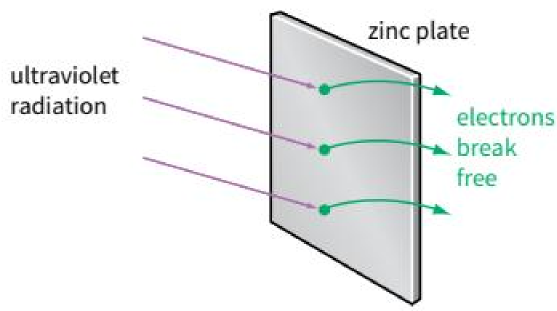

Figure 6: The photoelectric effect. When a photon of ultraviolet radiation strikes the metal plate, its energy may be sufficient to release an electron.

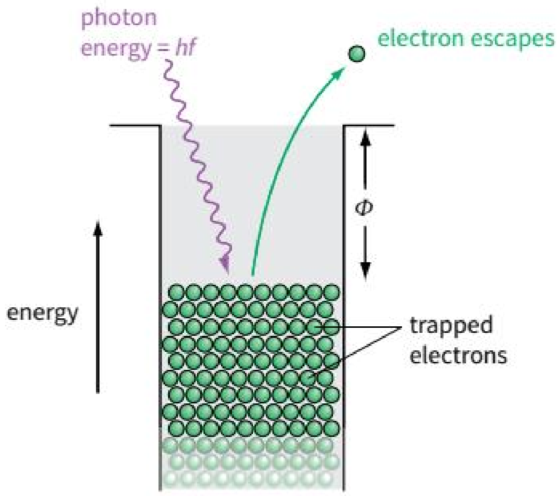

Figure 7: A single photon may interact with a single electron to release it.

Einstein did not picture electromagnetic waves interacting with all of the electrons in the metal. Instead, he suggested that a single photon could provide the energy needed by an individual electron to escape. The photon energy would need to be at least as great as Φ. By this means, Einstein could explain the threshold frequency. A photon of visible light has energy less than Φ, so it cannot release an electron from the surface of zinc.

When a photon arrives at the metal plate, it may be captured by an electron. The electron gains all of the photon's energy and the photon no longer exists. Some of the energy is needed for the electron to escape from the energy well; the rest is the electron's kinetic energy.

Now we can see that the photon model works because it models electromagnetic waves as concentrated 'packets' of energy, each one able to release an electron from the metal.

Here are some rules for the photoelectric effect:

- Electrons from the surface of the metal are removed.
- A single photon can only interact, and hence exchange its energy, with a single electron (one-to-one interaction).
- A surface electron is removed instantaneously from the metal surface when the energy of the incident photon is greater than, or equal to, the work function Φ of the metal. (The frequency of the incident radiation is greater than, or equal to, the threshold frequency of the metal.)
- Energy must be conserved when a photon interacts with an electron.

- Increasing the intensity of the incident radiation does not release a single electron when its frequency is less than the threshold frequency. The intensity of the incident radiation is proportional to the rate at which photons arrive at the plate. Each photon still has energy which is less than the work function.

Photoelectric experiments showed that the electrons released had a range of kinetic energies up to some maximum value, k.e.max. These fastest-moving electrons are the ones which were least tightly held in the metal.

Imagine a single photon interacting with a single surface electron and freeing it. According to Einstein:

energy of photon = work function + maximum kinetic energy of electron

$$hf = \Phi + k.e.max$$

Or

$$hf = \Phi + (1/2)mv^2max$$

This equation, known as Einstein's photoelectric equation, can be understood as follows:

- We start with a photon of energy hf.
- It is absorbed by an electron.
- Some of the energy (Φ) is used in escaping from the metal. The rest remains as kinetic energy of the electron.
- If the photon is absorbed by an electron that is lower in the energy well, the electron will have less kinetic energy than k.e.max (Figure 8).

What happens when the incident radiation has a frequency equal to the threshold frequency fo of the metal?

The kinetic energy of the electrons is zero. Hence, according to Einstein's photoelectric equation:

$$hfo = \Phi$$

Hence, the threshold frequency f is given by the expression:

$$fo = \Phi/h$$

What happens when the incident radiation has frequency less than the threshold frequency? A single photon can still give up its energy to a single electron, but this electron cannot escape from the attractive forces of the positive metal ions. The energy absorbed from the photons appears as kinetic energy of the electrons. These electrons lose their kinetic energy to the metal ions when they collide with them. This

warms up the metal. This is why a metal plate placed in the vicinity of a table lamp gets hot.

Different metals have different threshold frequencies, and hence different work functions. For example, alkali metals such as sodium, potassium and rubidium have threshold frequencies in the visible region of the electromagnetic spectrum. The conduction electrons in zinc are more tightly bound within the metal and so its threshold frequency is in the ultraviolet region of the spectrum.

The table below summarises the observations of the photoelectric effect, the problems a wave model of light has in explaining them, and how a photon model is more successful.

Table3: Differencies of wave and photon model

Observation	**Wave model**	**Photon Model**
Emission of electrons happens as soon as light shines on metal	Very intense light should be needed to have immediate effect	A single photon is enough to release one electron
Even weak(low-intensity) light is effective	Weak light waves should have no effect	Low-intensity light means fewer photons, not lower-energy photons
Increasing intensity of light increases rate at which electrons leave metal	Greater intensity means more energy, so more electrons are released.	Greater intensity means more photons per second, so more electrons released per second
Increasing intensity has no effect on energies of electrons	Greater intensity should mean electrons have more energy	Greater intensity does not means more energic photons, so electrons cannot have more energy
A minimum threshold frequency of light is needed	Low-frequency light should work, electrons would be released more slowly	A photon in a low-frequency light beam has energy that is too small to release an electronc
Increasing frequency of light increases maximum kinetic energy of electrons	It should be increasing intensity, not frequency , that increases energy of	Higher frequency means energetic photons; so electrons gain more energy

	electrons	and move faster

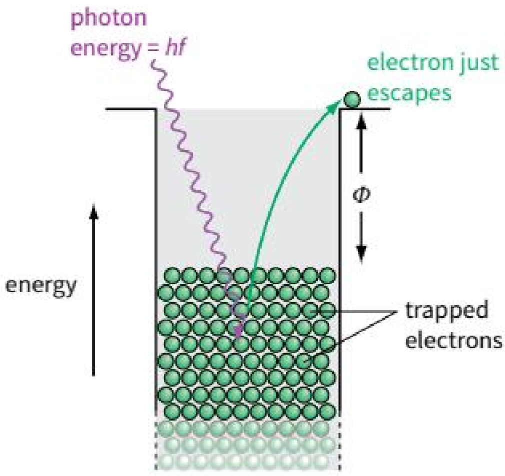

Figure 8: A more tightly bound electron needs more energy to release it from the metal.